Capacitive reactance

We find that when we apply a sine wave to a capacitor an interesting thing happens. Remember when we were discussing capacitors that it took time to charge and discharge a capacitor? This has a significant effect in regards to AC signals. Capacitors oppose changes in voltage by drawing or supplying current as they charge and discharge to the ever changing voltage level. The term for this is capacitive reactance. If we plot AC voltage and current across a capacitor we get the result shown in the graph below. We have added plus and minus signs to indicate voltage and current direction. Note that current in a minus direction does not mean it is lower in amplitude – it means it is going in the opposite direction than the plus direction.

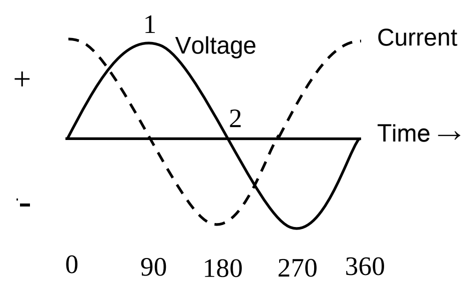

Fig. 15-6

This looks confusing at first but with a little analysis we can see that it is very logical. Let's take the case where voltage is at a peak (location #1 above). The capacitor reacts against a **change** in voltage – thus when voltage is at a peak there is no voltage change and so the current is zero. Similarly when the voltage is changing the most (where is crosses the zero line) is where current is strongest. You can see that current lags voltage by ninety degrees.

This also makes sense in regards to the equation we briefly looked at in the DC section.

$$i = C\,\frac{dv}{dt}$$

Where:
i = instantaneous current
C = capacitance
dv/dt = change of voltage over change in time

In this formula we have used an expression from calculus, dv/dt. For our discussion we can just think of this as meaning the change of voltage over time. Now we have introduced the variable of time. Previously when we were working with DC it didn't make any difference when the measurement was taken – now it does make a difference.

At the peak of the cycle (location #1), the change in voltage is zero, so at that instant, the current is zero. Where voltage crosses the zero line (location #2), the change of voltage is high, so current is highest.

We call this effect capacitive reactance. Of course we expect this effect to be related to capacitance, but it has also been found to vary with frequency.

The Formula for Capacitive Reactance

$$Xc = \frac{1}{2\pi f C}$$

where;
Xc = capacitive reactance (Ohms)
f = frequency (Hz)
C = capacitance (Farads)

Note that the reactance term Xc is in Ohms. This can be confused with resistance, but just remember that the use depends on whether the context is for a resistor or for impedance.

Examples:

Let's assume a 22uF capacitor is in the circuit with source on the left and a load on the right hand side as shown in the figure below. We are passing an AC signal of 200 Hz through the capacitor. There is a return ground implied.

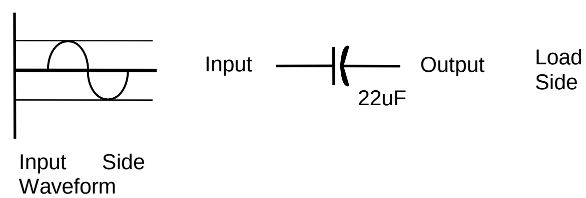

Fig. 15-7

First let's substitute 6.28 for 2π
Using our formula
Xc = 1/(6.28 X f X C)
Xc = 1/(6.28 X 200 X 22 X 10^{-6})
Xc = 1/27.6 x 10^{-3}
Xc = .036 x10^{3} = 36 Ohms

Now let's see what we get when we try to pass a much higher frequency of 1MHz

Now
Xc = 1/(6.28 X 1 X 10^{6} X 22 X 10^{-6})
Xc = 1/(138) = .007 Ohms

So we can see that the fairly large capacitor can pass both of these frequencies. By inspection, since capacitance and frequency are on the denominator (bottom) of the equation, higher frequencies and higher capacitance result in lower impedance.